PERMUTATION & COMBINATION

Complete SSC CGL Master Notes

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1. FUNDAMENTAL PRINCIPLE OF COUNTING

Basic Concepts

Fundamental Principle of Counting:
If one operation can be performed in 'm' ways and following that a second operation can be performed in 'n' ways, then both operations can be performed in m × n ways.

Multiplication Principle:
If there are n operations and operation 1 can be done in m_1 ways,
operation 2 in m_2 ways, ..., operation n in m_n ways,
then all operations together can be done in $m_1 \times m_2 \times \ldots \times m_n$ ways

Example: How many 2-digit numbers can be formed from digits 1,2,3,4?

Solution:

- Tens place: 4 choices (1,2,3,4)
- Units place: 4 choices (1,2,3,4)
- Total numbers = 4 × 4 = **16**

Addition Principle

Addition Principle:
If one operation can be performed in 'm' ways and another operation can be performed in 'n' ways, and both operations cannot be performed together, then either of the operations can be performed in m + n ways.

Example: From 5 Hindi and 4 English books, how many ways to choose one book?

Solution:

- Choose Hindi book: 5 ways
- Choose English book: 4 ways
- Total ways = 5 + 4 = **9 ways**

2. FACTORIAL NOTATION & BASIC FORMULAS

Factorial Definition

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n! = n × (n-1) × (n-2) × ... × 3 × 2 × 1
0! = 1
1! = 1
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Important Values:
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040
8! = 40320
9! = 362880
10! = 3628800

Permutation Formulas

Permutation of n different objects taken r at a time:

$^{n}P_{r} = n! / (n-r)!$

Special Cases:

$^{n}P_{n} = n!$

$^{n}P_{1} = n$

$^{n}P_{0} = 1$

Example: Find $^{5}P_{2}$ and $^{7}P_{3}$

Solution:

- $^{5}P_{2}$ = 5! / (5-2)! = 5! / 3! = (5×4×3×2×1)/(3×2×1) = 5×4 = **20**
- $^{7}P_{3}$ = 7! / (7-3)! = 7! / 4! = (7×6×5×4×3×2×1)/(4×3×2×1) = 7×6×5 = **210**

3. COMBINATION FORMULAS & CONCEPTS

Combination Definition

Combination is selection of objects where order doesn't matter.
Permutation is arrangement of objects where order matters.

Combination of n different objects taken r at a time:

$${}^nC_r = n! / [r! \times (n-r)!]$$

Special Cases:

$${}^nC_n = 1$$
$${}^nC_1 = n$$
$${}^nC_0 = 1$$
$${}^nC_r = {}^nC_{n-r}$$

Combination Properties

Important Properties:

1. ${}^nC_r = {}^nC_{n-r}$
2. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
3. ${}^nC_r = {}^nC_s \Rightarrow r = s \text{ or } r + s = n$
4. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

Example: Find ${}^8C_3$ and verify ${}^8C_3 = {}^8C_5$

Solution:

- ${}^8C_3 = 8! / (3! \times 5!) = (8\times7\times6)/(3\times2\times1) = 56$
- ${}^8C_5 = 8! / (5! \times 3!) = (8\times7\times6)/(3\times2\times1) = 56$
- Verified: ${}^8C_3 = {}^8C_5 = \mathbf{56}$

4. PERMUTATION VS COMBINATION - KEY DIFFERENCES

Comparison Table

Aspect	Permutation	Combination
Meaning	Arrangement of objects	Selection of objects
Order	Order matters	Order doesn't matter
Formula	${}^nP_r = n!/(n-r)!$	${}^nC_r = n!/[r!(n-r)!]$
Result	Always greater than or equal to combination	Always less than or equal to permutation
Example	Forming numbers from digits	Selecting team members

When to Use Which?

Use Permutation when:
• Arranging people in a row
• Forming numbers from digits
• Arranging letters to form words
• Any situation where order/position matters

Use Combination when:
• Selecting team members
• Choosing committee members
• Picking fruits from a basket
• Any situation where only selection matters

Example: From 5 people, select 3 for a committee vs arrange 3 in positions

Solution:

- Committee selection (order doesn't matter): ${}^5C_3 = 10$ ways
- Arrangement in positions (order matters): ${}^5P_3 = 60$ ways

5. PERMUTATION OF ALIKE OBJECTS

Objects with Repetition

Permutation of n objects with p alike of one kind, q alike of second kind, r alike of third kind:

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Number of permutations = n! / (p! × q! × r!)
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Example: How many permutations of letters in "BANANA"?

Solution:

- Total letters: 6
- B: 1, A: 3, N: 2
- Permutations = 6! / (1! × 3! × 2!)
- = 720 / (1 × 6 × 2) = 720 / 12 = **60**

Word Formation Problems

Example: How many words can be formed from "MISSISSIPPI"?

Solution:

- Total letters: 11
- M: 1, I: 4, S: 4, P: 2
- Permutations = 11! / (1! × 4! × 4! × 2!)
- = 39916800 / (1 × 24 × 24 × 2)
- = 39916800 / 1152 = **34650**

6. CIRCULAR PERMUTATIONS

Circular Arrangement Formula

Number of circular permutations of n distinct objects:

- When clockwise & anti-clockwise are different: $(n-1)!$
- When clockwise & anti-clockwise are same: $(n-1)!/2$

Example: 5 people around a circular table

Solution:

- Clockwise & anti-clockwise different: (5-1)! = 4! = 24
- Clockwise & anti-clockwise same: 4!/2 = 24/2 = 12

Necklace & Bead Problems

Example: 6 different beads to form a necklace

Solution:

- In necklace, clockwise & anti-clockwise are same
- Arrangements = (6-1)!/2 = 5!/2 = 120/2 = **60**

7. COMBINATION APPLICATIONS

Selection Problems

Example: From 7 men and 5 women, select 4 people with at least 2 women

Solution:

- Case 1: 2 women + 2 men = ${}^5C_2 \times {}^7C_2 = 10 \times 21 = 210$
- Case 2: 3 women + 1 man = ${}^5C_3 \times {}^7C_1 = 10 \times 7 = 70$
- Case 3: 4 women + 0 men = ${}^5C_4 \times {}^7C_0 = 5 \times 1 = 5$
- Total = 210 + 70 + 5 = **285**

Geometry Combination Problems

Example: How many triangles from 8 points on a circle?

Solution:

- Any 3 points form a triangle (no 3 points collinear)
- Triangles = ${}^8C_3 = 56$
- **Answer: 56 triangles**

Example: How many diagonals in a 10-sided polygon?

Solution:

- Total lines joining any 2 vertices = ${}^{10}C_2 = 45$
- Subtract 10 sides
- Diagonals = 45 - 10 = **35**

8. SSC CGL PRACTICE PROBLEMS & SHORTCUTS

Important Shortcuts

Shortcut 1: ${}^nP_r = {}^nC_r \times r!$
Shortcut 2: ${}^nC_r = {}^nC_{n-r}$ (Use when r > n/2)
Shortcut 3: For word problems, count repetitions carefully
Shortcut 4: In circular arrangements, fix one position first
Shortcut 5: For "at least" problems, use complement method

Practice Problem Set

Problem 1: How many 4-digit numbers can be formed from 1,2,3,4,5,6 without repetition?

Solution:

- ${}^6P_4 = 6! / (6-4)! = 6! / 2! = 720 / 2 =$ **360**

Problem 2: In how many ways can 5 books be arranged on a shelf?

Solution:

- $5! = 5 \times 4 \times 3 \times 2 \times 1 =$ **120**

Problem 3: From 10 people, select 4 for a committee. How many ways?

Solution:

- ${}^{10}C_4 = 10! / (4! \times 6!) = (10\times9\times8\times7)/(4\times3\times2\times1) =$ **210**

Problem 4: How many words can be formed from "MATHEMATICS"?

Solution:

- Total letters: 11
- M: 2, A: 2, T: 2, H: 1, E: 1, I: 1, C: 1, S: 1
- Permutations = $11! / (2! \times 2! \times 2!)$
- = 39916800 / 8 = **4989600**

SSC CGL Pattern Analysis

Topic	Frequency	Difficulty	Marks
Basic Permutation	High	Easy	1-2
Basic Combination	High	Easy	1-2
Word Formation	Medium	Moderate	2
Circular Arrangement	Low	Moderate	2
Geometry Combination	Low	Hard	2

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